PROBLEM SOLVING IN MATHEMATICS EDUCATION IN ITALY: DREAMS AND REALITY

Paolo Boero, Carlo Dapueto

Dipartimento di Matematica, Università di Genova

boero@dima.unige.it, dapueto@dima.unige.it

Drawing on personal experiences in in-service teacher training and curricular innovation in Italy, this paper addresses some questions relevant to mathematics education in the specific area of problem solving research. What are the effects of research results on national programs and curricula? To what extent, and how, are these assimilated in the school system? The analysis of some specific aspects of the evolution of the Italian situation in the last thirty years will suggest some possible answers suitable for comparison with other countries.

Introduction

Reflecting on problem solving as an object of the teaching and learning of mathematics in a given country offers many opportunities to deal with relevant issues in mathematics education, beyond the importance of problem solving in mathematics and in mathematics education. Indeed problem solving is a paradigmatic area of mathematics education as regards the evolution of the relationships among state-prescribed curricula, teachers' educational choices and expectations and textbooks features. In particular, reflecting on problem solving in a given country can allow a better understanding of the nature of the constraints that prevent the educational system taking into account research results that could improve the quality of teaching and learning. From another point of view, mathematical problem solving has been one of the relevant areas of mathematics education where researchers have felt (and still feel) the necessity of integrating competencies inherent in the reflection on the nature of mathematics with competencies deriving from psychology and other sciences of education.

Having these ideas in mind, in this paper we will try to show how teaching and learning difficulties concerning problem solving have influenced the evolution of research performed in Italy on this subject in the last thirty years, and how national prescriptions, textbooks and teachers have taken national and international results of problem solving research into account.

We will see how in Italy problem solving became a specific and important subject of research in mathematics education around the eighties, due to different motivations: reflections about the nature of mathematical activities, difficulties met by teachers in this specific area, changes brought in National Programs, and influences from international research. In more recent years, research on problem solving has addressed more specialized or related topics (e.g. affective issues related to problem solving, and specific kinds of problem solving such as conjecturing and proving on one side, and algebraic problem solving on the other). We will refer to some Italian research contributions that can be put under the cover of "problem solving research" showing the evolution of research trends related to the international context and national needs.

Until now, the influence of Italian (as well as international) problem solving research on textbooks and classroom practice in Italy has been extensive at a surface level (also due to prescriptions of National Programs*). However in the reality of classroom activities "exercises" still outnumber "problems" and the "broadcasting style" of lectures has not been modified. We will present some typical features of the situation, and discuss multiple reasons for it.

On the assessment side, this situation can partly explain while the PISA test (see: http://unesdoc.unesco.org/images/0014/001461/146184e.pdf) revealed a deficit in problem solving skills in Italian students. However there are also other reasons for it: in particular, the final maturity examination for scientific high school students (Grade XIII) is apparently based on problem solving, but most of those "problems" are "exercises" similar to those proposed in previous years. We will discuss the influence of the maturity examination on teachers' problem solving activities in the classroom, and on textbooks for scientific high school, as well as their side effects on the teaching of mathematics at previous levels of schooling and in the other parallel high schools.

This paper is entitled "Problem Solving in Mathematics Education in Italy: Dreams and Reality".

The "dreams" concern mathematics educators who, in the last thirty years, have tried to offer (through the connection with international research and their own contributions) tools and results to improve teaching and learning in this area. The "reality" concerns what happened and happens in the school system: a reality that is very far from dreams. The challenging task for us as researchers is to understand the reasons for the failure of the effort to improve teaching and learning of problem solving in Italian classrooms, and to try to make realistic hypotheses about how to overcome the situation.

Why Problem Solving Research in Italy

We can say that the origin of mathematics educators' interests in problem solving in Italy in the seventies largely depended on the reaction to the "modern mathematics" wave that reached Italy during the previous decade through some books (like OCDE,1961; Choquet et al.1961; and then Dienes,1966) and initiatives of the Ministry of Education (in-service training). In Italy, "modern mathematics" mainly meant teaching of structural aspects of basic mathematical concepts at the beginning of primary school (e.g. relations, approach to natural numbers through correspondences between sets, etc.), and early teaching of mathematical structures (e.g. groups and linear algebra) in

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*In Italy (like in other European countries) until the end of the last century the content and main methodological guidelines at every school level was established through National Programs issued by the Minister of Education, usually after the work of an experts' commission (including experts of the Ministry and experts in the teaching of the disciplines). The situation is similar to that of other European continental countries (like France), but in Italy the prescriptions have always been looser than in the other centralized countries, and also the control on their implementation in the schools has always been less strict.

secondary school.

In Italy, the "modern mathematics" wave produced, during the sixties and seventies, a large debate about how to improve the teaching and learning of mathematics. During the sixties, several mathematicians (like Ugo Morin, Luigi Campedelli and Francesco Speranza) had worked as mathematics educators to promote the diffusion of "modern mathematics" ideas, while other mathematicians (like Bruno De Finetti) and mathematics educators (like Emma Castelnuovo, a junior secondary school teacher well known at the international level: see Castelnuovo, 1963) reacted to it. Hans Freudenthal (see Freudenthal, 1973) and other mathematicians (like René Thom: see Thom,1973) and mathematics educators at the international level provided a basis for those Italian mathematics educators who were willing to improve the teaching and learning of mathematics in school, but who did not accept the epistemological and educational premises of the "modern mathematics" movement. The debate between supporters of and fighters against the "modern mathematics" perspective brought into evidence some peculiar characters of mathematics that highlighted the relevance of mathematical problem solving as a crucial area of the teaching and learning of mathematics. Indeed, let us assume that mathematics cannot be reduced to mathematical objects and structures, but that it also includes mathematical activities aimed at describing and interpreting "real world" phenomena (see Castelnuovo, 1963; Freudenthal, 1973), thus providing mathematical concepts with referential meaning (for a more recent framing, see Vergnaud, 1990). According to this assumption, problem solving becomes the core of the teaching and learning of mathematics, because problem solving provides students with the opportunity of engaging in meaningful mathematical activities that show them the necessity of mathematical tools to solve problems and allow developing the mastery of those tools as mathematical objects.

The national programs issued in 1979 for junior secondary school (grades VI-VIII) represented a compromise between those members of the National Commission who wrote the programs, who were supporters of the teaching of mathematical structures (like Francesco Speranza) and those members who were supporters of the constructive character of problem solving activities (including elementary mathematical modeling activities) in school (like Emma Castelnuovo). In particular, in the *"goals"* list we read *"Recognizing variable and invariant properties, analogies and differences"* and *"Arranging different matters into the same logical scheme",* but also *"Posing problems and generating solutions."* One of the *"Themes"* is *"Functions and structural analogies" ("To recall, compare and synthesize the concepts of relation, map, function, composition law arising from different areas")*, but another *"Theme"* is *"Problems and equations."* including *"To single out meaningful data and variables in a problem situation. To solve it by means of different procedures (flow charts, formulation and calculation of arithmetical expressions, etc.)."* It was the first time in the history of Italian compulsory education that problem solving entered as an important "content" in the national programs.

We find a larger extension for problem solving, with more detailed prescriptions, in the National Programs for elementary school (grades I-V) issued in the year 1985. Those programs were prepared by a long debate involving many mathematics educators. In 1983, a workshop in Trento had offered the members of the National Commission, who were writing the new Programs, the opportunity to meet with some outstanding foreign experts (E. Fischbein, K. Hart, I. M. Moser, G. Vergnaud). In those programs Problem Solving (mainly intended as *"Word problem solving"*) was the first of the five areas of mathematics education (the other were: *Arithmetic; Geometry and Measuring; Logic; Probability, Statistics and Information Technologies*). Concerning *"Problems"*, we read:

> *"Mathematical thought is characterized by the activity of the resolution of problems, and by this way it fits the child's tendency to ask questions and look for answers. Consequently basic mathematical notions should be founded and constructed starting from concrete problem situations, which derive from real experiences of the child and also offer the opportunity to ascertain his/her previous mathematical attainments, what tools and solving strategies he/she uses, and what difficulties he/she meets.*
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> *It is also important to avoid working in a confused and not well ordered way, and to move towards a progressive organization of knowledge.*
>
> *Aims:*
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> *- to translate elementary verbally expressed problems into mathematical representations, by choosing suitable operations; then to find the solutions and correctly interpret the results; vice versa, to provide a meaning for given mathematical representations;*
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> *- to find problem situations in experience and study domains, to formulate and justify solving hypotheses by means of suitable mathematical (either arithmetical or not) tools;*
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> *- to solve problems that have only one solving procedure or solution, and problems that allow different, but equally acceptable, solutions;*
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> *- to single out the lack of data which are essential for the solution of a problem and, if this is the case, to complete the set of data; to recognize the presence of too many, or contradictory, data in a problem, and to establish whether it is possible to solve it or not."*

The importance of problem solving in the National Programs issued in 1985 is enhanced by prescriptions and suggestions contained in other chapters: for instance, in the chapter "Arithmetic" we read: *"The development of calculation skills should be based on concrete models closely related to problem situations."* with a related goal: *"To perform the four arithmetical operations with natural and decimal numbers, and to understand the meaning of the algorithms"*.

A Case of Interest: Problem Solving Research in Italy, 1986-1992, and the Evolution of the Teaching of Mathematics in Grades I-V

The new Elementary School National Programs issued in 1985 provoked an interesting evolution in the effort of some Italian mathematics educators: in the previous decade they had tried to show (through pilot experiments) that it was possible teaching mathematics through problem solving activities, and to get a large consensus (based on epistemological reflections and concrete experiences) on the importance of problem solving in the mathematics classroom. Once problem solving had become a relevant chapter in the national programs, it was necessary to support teachers with tools to plan, manage, interpret and evaluate problem solving activities in the classroom. A unique occasion for this support was offered by the Italian Ministry of Education, which organized a nationwide long term project for the in-service teacher training of elementary school teachers (1986-1990). The project was implemented at the regional level (there are 19 regions in Italy) for the preparation of materials (mostly, booklets and videotapes for teachers), the choice of "experts" and the preparation of tutors. Most of the Italian mathematics educators influenced this Project in some way: directly, through their contributions to the writing of booklets and video-recordings of their lectures; indirectly, through experimental research and theoretical framing of problem solving in the classroom.

Twenty years after that period, we can say that the engagement of Italian mathematics educators in problem solving research has had greater impact on the evolution of Italian research in mathematics education than on the teaching and learning of mathematics in Italian schools.

The importance of problem solving research for the evolution of Italian research in mathematics education can be related to the previous situation of Italian research in this field. Up through the mid-eighties, Italian "research" in mathematics education had been mostly dedicated to the implementation of innovative ways of teaching mathematics in the classroom, with special attention paid to the cultural quality of the content taught, and its organization into gradual (frequently, spiral-shaped) sequences of tasks and explanations. The focus on those aspects of teaching occurred because most mathematics educators were mathematicians who engaged in mathematics education for social and cultural reasons. However, focusing on only those aspects had an important consequence: even though some Italian mathematics educators took part during the sixties and seventies in international CIEAEM and ICME meetings, their interests in the debates mainly concerned epistemological and cultural aspects of the teaching of mathematics. Dealing with problem solving induced some Italian mathematics educators to take relevant cognitive issues into account, according to contemporary research in the field, and this went far beyond the initial reference to Polya's seminal work (see Polya, 1954). Indeed the implementation of the National Programs in the classrooms had put into evidence some difficulties, which concern (according to present terminology): linguistic and semiotic aspects of problem solving; those forms of hypothetical reasoning, which are needed in problem solving and problem posing; mental dynamics that relate the use of memory to anticipation and planning; the influence of contextual factors on students' problem solving activities; specific areas of problem solving (proportionality problems); and the relevance of the didactical contract (see Brousseau, 1997). Interesting data concern Italian participation to PME (the International Group for Psychology of Mathematics Education, founded during the 1996 ICMI Congress in Karlsruhe): until PME-X, only two Italian mathematics educators had taken part in PME conferences. Starting with PME-XI (1988), a growing number of Italian mathematics educators took part in the PME Conferences, with an increasing number of accepted research reports. In PME-XII, PME-XIII, PME-XIV and PME-XV, most Italian contributions concerned issues related to problem solving and specific teaching and learning difficulties met in classrooms (like linguistic and representational aspects of problem solving activities, context-dependence of problem solving strategies, mental dynamics and the production and management of hypotheses in problem solving: see Boero, 1988, 1990; Boero & Shapiro, 1992; Ferrari, 1989, 1990, 1992; Arzarello, 1989; Bondesan&Ferrari, 1991; Malara & Garuti, 1991; Garuti & Boero, 1992). We can observe that some of those papers contained germs of ideas that Italian research in mathematics education has developed in the following years: proportionality problems (see Malara & Ponzi, 2003); the role of hypothetical reasoning in mathematical modeling and problem solving (see Ferrari, 1993); the functions of natural language in mathematical activities (see Ferrari, 1996; Boero, Douek, Ferrari, 2002) and semiotic aspects of problem solving (see Arzarello et al., 2006; Bazzini, 2002; Bartolini Bussi & Boni, 2003; Ferrara & Robutti, 2002); the role of the context (meant as "field of experience") as a resource in problem solving and in the construction of mathematical knowledge (see Boero, 1993; Boero et al., 1996; Bartolini Bussi et al, 1999; Bonotto, 2001a, 2001b); the early approach to algebra through word problem solving activities (see Malara, 1999); mental dynamics in word problem solving (see Guala & Boero, 1999), in conjecturing and proving (see Arzarello et al, 2002; Boero et al., 1996, 1998) and in algebraic problem solving (see Boero, 2001).

As remarked before, while the engagement of Italian mathematics educators in the field of mathematical problem solving had some relevant implications for the development of Italian research in mathematics education, it had little impact on the practice of problem solving activities in the classrooms. We can ask ourselves why this happened.

As mentioned above, the national long term project for the compulsory in-service training of elementary school teachers on the National Programs issued in 1985 provided Italian mathematics educators with an extraordinary opportunity to intervene, at the regional level, in the preparation of all elementary school teachers. Our estimation is that mathematics educators strongly engaged in the implementation of the national project in at least 9 regions (including the biggest ones) out of 19. Some factors can explain their limited impact even in those regions (particularly as concerns problem solving).

One factor was probably the initially weak nature of their research results and preparation in the area of problem solving; the first (and partial) survey in Italian on problem solving research at the international level was published only in 1987 (see Boero & Ferrari, 1987); through that survey, Italian mathematics educators came into contact with contemporary international research on problem solving and relevant sources (like the book of Schoenfeld, 1985). The first papers dealing with problem solving research in Italy have been published in the proceedings of international conferences and international journals (see Boero, Ferrari & Ferrero, 1989) at the end of the eighties, while the activities of teachers' preparation had been planned in the year 1986.

Another, more relevant factor was the strong impact (on teachers' practices) of textbooks published to support the implementation of the National Programs in the school system. Textbooks appeared to quickly conform to the "Ministry New Programs" (as they were called in the schools). But if we try to check what "conforming" meant, we see that in most cases (including the most widely diffused textbooks) the spirit of the new Programs was completely lost. The prescription *"To single out the lack of data which are essential for the solution of a problem, and, if this is the case, to complete the set of data"* resulted in a chapter of the textbook dedicated to it, including a set of exercises of increasing difficulty and complexity aimed at finding and completing lacking data (frequently, the exercise did not contain the request to solve the problem!). The prescription *"To translate elementary problems expressed verbally into mathematical representations"* resulted into a set of exercises aimed at choosing suitable representations, from a set of pre-assigned representations, for stereotypical problem situations. The prescription *"The development of calculation skills should be based on concrete models closely related to problem situations."* with a related goal: *"To perform the four arithmetical operations with natural and decimal numbers, and to understand the meaning of the algorithms"* resulted into a more detailed explanation of "how" an algorithm (for the computation of a subtraction or a multiplication or a division) works, but no explanation was provided about "why" it works, and no connection was made with problem situations.

Why did the most widely diffused textbook provide the teachers with those kinds of interpretations of the New National Programs? Probably, the lack of preparation of teachers on fundamental cognitive and didactical issues concerning the teaching and learning of mathematics was one of the main reasons for it. Indeed, it is necessary to consider the fact that most Italian teachers (of all school levels) in that period had not received any institutional preparation (in pre-service training) on the teaching and learning of mathematics. In particular, until the end of the last century, elementary school teachers' preparation was usually acquired in a 4 or 5 years (from grade IX to grade XII or XIII) high school specially devoted to the preparation of elementary school teachers; in that school there was only room for essential information about theories of developmental psychology and some general pedagogical principles. The authors and editors of textbooks were conscious of the teachers' level of preparation; those mathematics educators who helped write textbooks for children or booklets for teachers encountered big difficulties when they tried to convey research-inspired ideas or translate principles, suggestions and prescriptions of national programs into coherent materials for classroom activities. An interesting anecdote concerns one of the authors of this paper: he engaged in the writing of monthly contributions ("guidelines") on the teaching of mathematics in the elementary school *"according to the new Programs"* under one of the most important and advanced editors of such professional tools. In one of the monthly papers, he wrote that the prescription *"to single out the lack of data which are essential for the solution of a problem, and, if this is the case, to complete the set of data"* should not be implemented only through a set of tasks with the explicit request to find lacking data, but it was good also to engage students in meaningful activities of problem solving (possibly related to their ordinary life experiences) in which they might realize by themselves that there are cases in which some data where lacking. The editor of the journal told him that teachers would have not been able to manage such an open situation and deal with their students' failure in solving the problem! He wrote that *"students, as well as their parents, could think that the teacher is not aware of the impossibility of solving that problem, and this is not acceptable for the teachers"*. Even suggestions concerning students' free search for meaningful mathematical representations of problems were censored, with the motivation that *"teachers are not able to manage a classroom situation in which a plurality of representations have been produced"*. The editor suggested proposing tasks where the aim was to choose (between pre-assigned symbolic representations) the "correct" one, or to adapt a given representation (e.g. $a+...=b$) to the given arithmetic situation by choosing the appropriate numerical values of a and b.

Apart from these reasons, there are other related facts that can explain why the engagement of mathematics educators in problem solving research and in the National Project for in-service teacher training did not change the situation of the teaching of problem solving in Italian elementary schools. Drawing on our personal experiences of in-service training and work with teachers in planning and experimenting didactical innovations, we have realized that most teachers are not willing to move towards a genuine activity of problem solving in the classroom because it does not fit their model (frequently, an unconscious model!) of teacher in the mathematics classroom. This interpretation is not new in mathematics education research (see Jaworski, 1998; and several contributions in Krainer, Goffree & Berger, Eds., 1999), but in the case of the National Programs issued in 1985 we have found specific relationships between some prescriptions and the explicit declarations of some teachers against them. One of the authors of the present paper has collected numerous examples in which a teacher takes the initiative to express his/her refusal of the spirit of the National Programs and the other teachers do not react (thus showing that they agree on the refusal).

In particular, these sentences faithfully represent positions that seemed (and still seem) to have a wide consensus among mathematics teachers in primary school:

> *"Mathematics teaching is not like the discussion of a tale, children must concentrate and learn precise procedures and ways of reasoning; if I leave them free in a mathematics situation, it is difficult to re-establish a productive climate for learning mathematics";*
>
> *"Many children are not able to produce good solutions in genuine mathematical problem solving; they are only able to reproduce the ways of reasoning that I present to them";*
>
> *"Many children suffer from encountering uncertain situations, their age is not suitable for open situations; it is true that they ask questions, but I see that they are happy when I give the answer, while I see that they are not willing to find the answer by themselves".*

These positions call into question the possibility of developing "authentic" problem solving skills in elementary classes according to different reasons: the unavoidable break of the didactical contract (cf Brousseau, 1997); the children's cognitive potential; and considerations related to the children's affective dimension. Shortly, "authentic" problem solving would not be possible in the classroom for obstacles inherent in students' behavior, potential and ways of thinking. However, in some cases we were able to have a deeper discussion and we have found that there are also reasons related to teachers' expectations about their place in the classroom and their relationships with students' parents (see Brousseau, 1997). Sometimes, the teachers' sentences during a hot debate remind us what the editor of the monthly journal for teachers wrote to one of the authors: *"students, as well as their parents, could think that the teacher is not aware of the impossibility of solving that problem, and this is not acceptable for the teachers".* In other cases we have found some evidence for the fact that teachers do not control the mathematical and/or the cognitive background of the students' behaviors (thus they stick to the procedures and formalisms that they manage, and the fear to move to an unknown ground prevents them from engaging students in "free" problem solving activities). We have collected sentences that reveal how and why the teachers are uneasy with "free" problem solving. Here there is an excerpt of a discussion among teachers (Angela is a teacher-researcher, who is leading a working group on problem solving during an in-service teacher training activity).

> Rita: *"If you leave the students free to write or to say what they want, you cannot manage the situation, the situation become chaotic"*
>
> Angela: *"But you can make a selection of what they write, and ask to discuss those solutions"*
>
> Anna: *"But how to make a selection? When I read what they write in some cases I cannot understand what they think, in other cases I cannot realize if it is a good idea or not"*
>
> Rita: *"And how to choose the best solution and convince them that it is the best solution? I have my solution, sometimes I have the doubt that a child has produced a very good idea, but I am not sure about it. It is better to teach my solution!"*

Such an excerpt seems to confirm and even expand what that editor of a monthly journal for teachers wrote to one of the authors of this paper: *"teachers are not able to manage a classroom situation in which a plurality of representations have been produced."* Teachers seem unable to manage not only the case of a plurality of representations, but also the case of a plurality of solutions!

The situation described in this section challenges mathematics educators on the fundamental grounds: how to prepare teachers in the pre-service teacher training courses (since 1999 primary school teachers are prepared through a four-years university curriculum; we will discuss this issue in the last section of this paper); and how to intervene in the in-service teachers' training (see Boero, Dapueto & Parenti, 1996).

From the research point of view, the difficulties met with ordinary teachers in the implementation of the 1985 National Programs for elementary school was one of the main motivations for the development of research on teachers' and students' beliefs in the following two decades, keeping international research into account (Schoenfeld, 1989 and Mc Leod, 1992 were the initial reference papers; see Di Martino & Zan, 2003 for a recent typical Italian contribution). It is of interest for this paper to observe that Furinghetti & Pehkonen (2000) showed (in their comparative study between Finnish and Italian students) how the biggest differences between the two countries were found in the items concerning the use of trial-and-error strategies and the possibility that students solve mathematical problems on their own.

Another Case of Interest: Problem Solving in National Programs and Schools at the High School Level, or: How Teaching is Shaped by the Maturity Examination "Problems"

An important starting point for considering the evolution of National Programs at the high school level in Italy is the lack of a comprehensive reform since the so-called "Gentile reform" in 1923, which established a precise hierarchy between the different secondary schools (classic studies-oriented high school, scientific-oriented high schools, technical institutes and professional schools) according to both the relative weight of humanistic, scientific, technical and professional disciplines, and their orienting function towards further studies and professions. At the beginning, from classic-oriented high school it was possible to access every University curriculum, while the access to scientific and technological faculties was the privileged outcome for scientific-oriented high school, and no access to University was possible for people with a diploma in professional schools. The lack of an systematic reform (which was due to the lack of a political consensus on such a delicate and complex matter within the different political coalitions that lead Italy after the second world war) limited the Parliament and the Ministries of Education to partial interventions: in particular, for social and political reasons, access to University was progressively opened (since the sixties) far beyond the original design of the Gentile Reform, in an attempt to avoid social discriminations. Other interventions (at the level of the Ministry of Education) concerned National Programs, with a double aim: to take into account the evolution of culture in the disciplines taught at school, and to meet the needs inherent in the potential outcomes of the different schools and related preparation.

In particular, national programs for Scientific High Schools had been reformed by the Anglo-American occupants in Italy at the end of the Second World War; no systematic, coherent change was brought in the following five decades; there were only some limited changes in single disciplines (mathematics, physics, etc.).

The lack of a coherent design for new official curricula in Mathematics and the other disciplines, due to the lack of a general reform design at the institutional level, opened the possibilities of elaborations and experimentations in single disciplines in order to overcome the consequences of such inertia. Mathematicians who engaged in Mathematics Education first, tried to propose new programs for Mathematics during the sixties (the Frascati Programs), then they realized the impossibility of getting satisfactory changes at the institutional level, and some of them moved towards planning and experimenting with innovative curricular projects, during the seventies. A strict relationship was established between mathematicians and teachers engaged in those Projects, and we recognize now that it was one of the roots of present mathematics education research in

Italy, mostly characterized as "Research for innovation" (whose principles are presented in Arzarello & Bartolini Bussi, 1998; see also Malara & Zan, 2002), with a close partnership between university researchers and teachers-researchers (i.e. teachers who participate in the activities of the University research groups and not only perform the teaching experiments in their classrooms, but also share in motivating, planning and analyzing classroom activities).

The seminal cultural and experimental work of the Nuclei di Ricerca Didattica (Didactical Research Groups) led by University mathematicians (like Giovanni Prodi, Francesco Speranza, Vinicio Villani) was supported by the CNR (National Council of Research) with research funding and fellowships to prepare researchers in the field of Mathematics Education. The activities of the Didactical Research Groups influenced the projects and initiatives promoted by the Ministry at the national level in the following decades, in particular the Ministry programs of mathematics for the National Plan for Computer Sciences in Scientific High School issued in 1989 (after for years of in-service training) and the so called "Brocca Programs" issued in 1991. Both Programs were adopted in experimental high school sections, while the programs of the traditional Scientific High Schools remained substantially unchanged till 2003 (even if the Ministry invited teachers to take the "Brocca Programs" into account as references to update teaching of mathematics).

Coming to the theme of this paper, we can ask ourselves how problem solving was present in those programs and how prescriptions concerning problem solving were related to research on problem solving, on one side, and had influences in the traditional Scientific High School, on the other. We will consider only the "Brocca programs" (similar comments can be made for the other programs).

Differently from the Elementary School Programs issued in 1985, there is no specific section, or theme, for problem solving in the High School "Brocca Programs", which are organized according to mathematical content themes, although there are some explicit or implicit references to problem solving in the text. According to Freudenthal's views about horizontal mathematization and vertical mathematization, programs provide prescriptions and suggestions concerning the activities of problem solving inherent in internal mathematical modeling (e.g. using algebraic tools to solve geometrical problems) and in external mathematical modeling (e.g. using algebraic tools to solve a problem in physics), as well as in conjecturing and proving activities. The scientific debate at the international level on the relationships between problem solving and mathematical modeling (cf Blum & Niss, 1989) was influential on it, as well as the idea (shared by most Italian mathematics educators in that period) that problem solving had to be considered, at the high school level, not as a separate area of mathematics education, but as a way of organizing classroom activities on crucial mathematical subjects (like conjecturing and proving, modeling, etc.).

Even if there were good cultural reasons for that choice, the lack of a section in the programs specially devoted to problem solving was probably one of the reasons why the teaching of problem solving has not changed in high schools (including the experimental Sections adopting the "Brocca programs") in the last twenty years, and teaching of problem solving today is not very different from what it was fifty years ago.

We must say that, with respect to problem solving, our personal High School textbooks (resp. during the fifties and during the sixties) are not different from those adopted today by many teachers, although some changes were brought to the content according to the prescriptions of the “Brocca Programs” for experimental sections (e.g. additions of new chapters or sections concerning Logic, Programming, Statistics and Probability). In the reality of most textbooks and classroom practices in Scientific High Schools today, the activities of "problem solving" are functionally related (as forty or fifty years ago) to the preparation to the final examination, in which the time-invariant component in the last fifty years is represented by one or two "big problems" (typically, a "problem" concerning geometry to be dealt with algebraic tools and one Calculus "problem" concerning the study of an one-variable function with tools from Calculus). The problem solving curriculum followed in most high school classes can be described as a curriculum shaped by the need to solve the maturity examination "problems" at the end of high school. The nature of those problems is quite suitable for gradual, systematic preparation: for instance, a typical Calculus “problem” task requires students to study one function in one variable (using domain of definition, limits at the border of the domain, minima and maxima with the use of the first and second derivative, etc.), the intersection of the graph of the given function with the graph of another function, then the calculation of the area of the surface underlying the first graph between two of the points of intersection. Students are prepared for this routine through a step-by-step approach to it: they learn to find the domain of a function, typically given as a rational function or an algebraic expression under square root, then to calculate limits, derivatives, etc. The teaching and learning of elements of calculus in the last year of high school (18-19 years old students) is shaped by this sequence of tasks, which are the components of the “complex” final calculus “problem” of the maturity examination. Also, many preliminary activities (concerning functions, analytic geometry, etc.) in the previous years are related to the same aim: students learn to deal with elementary functions that are the components of the more complex function studied in the fifth year, while other important kinds of functions (e.g. piecewise-defined functions, functions containing an absolute value, etc.) have a marginal weight in the curriculum. Similar things can be said for the other “big problem” of the maturity examination, which concerns geometry: the problem frequently asks students to put some relationships between elements of a plane or space geometric figure into a one-parameter equation, to “discuss” the solutions of the equation (existence in the real field, positivity, etc) as they relate to the values of the parameter, etc. In Scientific High Schools, starting at grade IX, both the geometry curriculum and the curriculum on equations are shaped according to the needs of solving the above geometry "problem," with a sequence of increasingly demanding "problems" concerning plane and space geometry, and the solution (and discussion of the solutions) of equations (from linear equations to quadratic equations and other equations that can be reduced to quadratic equations: biquadratic equations, trigonometric equations, etc).

The room for open problem solving activities is very limited; very few teachers (and few textbooks) offer students the opportunity to engage in conjecturing and proving problems (e.g. in the field of elementary arithmetic: properties that can be conjectured through exploration in numerical cases and proved through the use of algebraic language). Sometimes the teaching of Physics (the same teacher teaches both Mathematics and Physics) offers the possibility of proposing open mathematical modeling problems, based on the use of first and second degree equations or trigonometric functions.

The introduction of elements of programming statistics and probability in the "Brocca Programs" might (according to the suggestions of the Programs) offer some opportunities of interesting open problem solving activities. Unfortunately, most teachers did not learn those subjects in their university curricula and their in-service preparation was not sufficiently long and deep. Thus, teachers do not feel secure in moving from routine tasks towards more demanding and open problem situations. Even the use of geometric or algebraic software (like Cabri or Derive), which is encouraged by some regional institutions of the Ministry and supported by a rather good quantity and quality of computers in each school, could result in a more lively and "creative" classroom mathematics, but again problems of teachers' preparation and the need for a relatively large amount of time for those activities reduces their potential impact in schools.

In Italy, research on problem solving in the last twenty years could offer some opportunities to move from the traditional ways of teaching mathematics in the Scientific High Schools to a more open and challenging approach to advanced mathematical content for students. In particular, past Italian research on conjecturing and proving with or without software at different school levels (see Arzarello, Bartolini Bussi & Robutti, 2002; Bartolini Bussi et al., 1999; Boero, Garuti & Mariotti, 1996; Boero, Garuti & Lemut, 1998; Mariotti, 2001) has produced interesting examples (accessible to teachers through written reports on teaching experiments and Internet) of how to approach theorems in school in a "natural" way, in particular as statements deriving from explorations and which can be proved by enchaining (in a deductive way) arguments produced during the exploration phase (see Italian contributions in Boero, Ed., 2006). Italian research on mathematical modeling in secondary school has shown how students can be involved in meaningful problem situations dealing with current subjects of interest in today's debates (growing of the human population, pollution, economics, etc.: see the next section). However it is hard for those ideas to penetrate school mathematics because they require time (time constraints are very strict, if a teacher wants to prepare all his/her students to pass the maturity examination!) and they require a change in the didactical contract and in the teachers' professional attitude.

The Situation in the Italian School in the Present Decade: From National Programs to Indications for Curricula in the New School System

Thus far we have focused our attention mainly on what happened in Italy some fifteen - twenty years ago; that period was a crucial time for the first important changes in Italian national programs after several decades, for the explicit introduction of problem solving issues in the Italian school, and for the development of Italian research in mathematics education (particularly in the field of problem solving). We have also considered what happened in Italy in the following years (particularly as concerns problem solving issues in elementary school and in the Scientific High Schools).

At the end of the previous century and at the beginning of the present decade (partly as a consequence of the NCTM standards issued in the year 2000, but mainly for reasons inherent in the necessity of upgrading previous prescriptions to the evolution of the Italian school system) the Ministry of Education promoted a large transformation of the ways of conceiving the relationships between prescriptions at the national level and their implementations in the school system. The idea of autonomous school "piani dell'offerta formativa (POF)" (in English: "school educational offer") was implemented in parallel with increasing administrative and financial autonomy for schools. For instance, principals of the schools ("presidi" and "direttori didattici") became "school managers", with increased responsibilities and salary. This reform was intended to better satisfy the needs of the different regions and local communities. It was also conceived in order to overcome the previous rigidity of the Italian school system (still shaped according to the Napoleonic model of the 19th century) and (for secondary schools) to escape the consequences of the impossibility of deciding on a comprehensive reform at the political level. The increased autonomy of schools should have been equilibrated by a National System of Evaluation (but it is still in a phase of controversial experimentation).

Starting from 2001 in Elementary and Lower Secondary School (grades I-VIII), some "orientations for the elaboration of school curricula" have been issued by the Ministry of Education. In the case of Mathematics, a Commission of the Unione Matematica Italiana (U.M.I., the Italian Mathematical Union, i.e. the professional association of mathematicians), including mathematicians, mathematics educators and representatives of the Ministry, as part of an agreement between the Ministry and

U.M.I., undertaken the task to prepare those "orientations". However, the broad documents elaborated by the Commission were "synthesized" when they became Ministry documents, losing some of the most innovative prescriptions. Currently, the teaching of mathematics in Italy has available the Ministry "orientations", the documents of the U.M.I. Commission, and some suggestions and examples (including evaluation of learning) for their implementation in schools (issued by the U.M.I. Commission).

The importance and visibility of problem solving grew up: in the Documents of the U.M.I. Commission, it is one of the seven big areas of the Orientations (they are: numbers and algorithms; space and figures; relations and functions; data and forecasting; arguing, conjecturing and proving; measuring; problem solving and problem posing) for all school levels.

Italian research activities in Mathematics Education have been very influential on the Documents of the U.M.I. Commission (the Commission included some researchers in Mathematics Education). In particular, in the Introduction to "Problem solving and problem posing" for Grades IX- XII (Orientations for New Curricula, 2003) we can read that *"in this area, the key word is mathematical model"*. This statement is clearly related to some research results that show how the practice of traditional word problem solving in school mathematics hardly matches the idea of mathematical modeling and mathematization (see Bonotto, 2001b, 2006); thus problem posing and problem solving are re-interpreted in terms of mathematical modeling (cf the idea of model elaborated in Dapueto and Parenti, 1999, which includes both internal modeling in Mathematics, and modeling of natural and social phenomena). In the same Introduction we see how the activities of problem posing and problem solving are conceived as highly demanding argumentative activities (with an explicit reference to Polya, 1954 on the epistemological ground), with clear connections with Italian research on linguistic and semiotic aspects of problem solving (see Arzarello et al., 2006; Boero, Douek & Ferrari, 2002; Ferrara & Robutti, 2002; Bartolini Bussi & Boni, 2003).

However in the present situation of evolution of the school system towards autonomy the impact of the U.M.I. Commission documents, and even the impact of the official Orientations of the Ministry, is not very strong. We can say that the real situation (both at the primary school level and in secondary school) is very similar to that outlined in the previous Sections for the past decade.

From Past Dreams to a Hope for the Future: Problem Solving in Teachers' Preparation.

Another, important and potentially more influential change was brought to the Italian school system at the end of the past decade: the teachers at all school levels must have an University professional preparation (at present, through a 4-years ad hoc curriculum to teach up to the grade V, and a 5+2 years curriculum including two years of professional preparation to teach in grades VI-XIII).

This change offered the possibility for Italian mathematics educators to engage systematically in teachers' preparation at the University level. Previous experience regarding teachers' difficulties in implementing the National prescriptions (and the indications coming from research on mathematics teachers' preparation: see Jaworski, 1998; Krainer et al., Eds, 1999) suggests that we should engage prospective teachers:

- in activities at the adult level, conceived according to present views on problem solving; prospective teachers must experience (as students) good models of teaching, where problem solving activities are important to build concepts, mathematize real world situations, and develop argument;

- in the management of problem solving activities in the classroom during their training stages in the school, with their tutors' help in order to learn how to interpret students' productions (by using Mathematics Education tools), use and compare them in the classroom, etc.

Both research at the international level (see Simon, 1994) and our elaborations on this subject (see Boero, Dapueto & Parenti, 1996) suggest establishing such a narrow relationship between prospective teachers' learning processes in mathematics, their reflections on their own difficulties, and classroom situations.

We do not yet have sufficient experimental evidence about the impact of these activities in teachers' University preparation on the teachers' ordinary work in the classrooms. However both research on teachers' preparation at the international level and some data collected by us in the last two years raise some hopes about the possibility of changes in teaching (including an increasing engagement in teaching of authentic problem solving) carried out in school by the new kind of teachers prepared during this decade. In particular, we are observing an interesting shift in the adoption of textbooks towards less traditional and closed-minded textbooks (usually supported by "new teachers" in their schools) and an increasing demand from them to connect with educational research teams on subjects like problem solving, the development of argumentative skills, a productive use of new technologies, etc.

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